NEW CONCEPT OF IMAGE RESTORING

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ABSTRACT

The high-resolution is required for various image and multidimensional signal applications. We propose a method of reconstruction of high-resolution images from several blurred images recorded by combined system (gathering several images by different imaging systems). It is based on the stabilizing algorithm developed by Dovnar of determination of the N-dimensional of the object by means of the linearly formed M-dimensional image. The stabilized algorithm allows reconstruction of the object from an image with a finite number of sampling from the square integrable function space. The algorithm is good for reconstruction of objects and signals from an image blurred by atmosphere, diffraction, moving, sampling, etc. artifacts. In addition, a correct application of the new method allows to obtain the superresolution. The validity of the proposed method is both theoretically and experimentally demonstrated. The method can be widely used for design of new systems, i.e. for remote sensing and researches of physical objects.

1. INTRODUCTION

The image restoration problem is the inverse problem and it consists in the mapping data from the blurred image to the actual object. At present the design of image gathering devices, and processing for image restoration are the intrinsic elements of nowadays system [1], [2].

The Fredholm integral equation can be used for solving this problem as ill-posed. The Wiener filter, which may be a most common image restoration algorithm [3], is accepted for comparing with new algorithms [4]. However it is not applicable for every system [5].

In this paper, we propose a concept of image restoration by a combined system (restoration from several images or several image gathering systems) [6]. The theoretic simulations and experiments were performed for reconstructing images, which were blurred by scattering medias and diffraction limited imaging systems. The use of the several images of the object with different kind of illumination makes it possible to transfer high spatial frequencies of the object beyond the diffraction limit practically without degradation. The contrast of the reconstructed image is greater considerably than that of the object reconstructed from one image.

The new concept can be applied for design of new devices for gathering, restoration and communication of image. For evaluation of the efficiency of systems designed the method of comparative information analysis [7], [8]. This method is based on calculations of the minimum mean square error of the estimation of the N-dimensional object [8]. The comparative information analysis takes into account the imaging conditions (point spread function (PSF), a position and a number of reading points of the image, a geometrical form of pixels of the staring focal-plane array devices, and also statistical properties of an object and noise) similarly to the Wiener filtration).

2. OPTIMAL RESTORATION ALGORITHM BY COMBINED SYSTEMS

2.1. Image restoration by a combined system

The process of the image forming by combined system is described by the Fredholm integral equation of the first kind:

$$\int_{-\vec{S}}^{\vec{S}} z(\vec{\xi})K(\vec{x},\vec{\xi},p)d\vec{\xi} = f(\vec{x},p) + \gamma(\vec{x},p) = F(\vec{x},p)\ , \qquad |\vec{x}| \le \vec{R}\ , \tag{1}$$

where $z(\vec{\xi})$ are unknown object characteristics, $\vec{\xi} = (\xi_1, \xi_2, ..., \xi_M)$, $d\vec{\xi} = d\xi_1 d\xi_2 ... d\xi_M$, $\vec{S} = (S_1, S_2, ..., S_M)$, $\vec{x} = (x_1, x_2, ..., x_N)$, $d\vec{x} = dx_1 dx_2 ... dx_N$, $\vec{R} = (R_1, R_2, ... R_N)$. The inequalities of a kind of $|\vec{x}| \le \vec{R}$ are denoted $|x_i| \le R_i$ for every i. The integer variable p is a number of an individual system. The right-hand part of Eq. (1) is known approximately: $f(\vec{x},p)$ is the exact value, $\gamma(\vec{x},p)$ is the value representation error (noise). The square integrable kernel of Eq. (1) $K(\vec{x},\vec{\xi},p)$ we define by the following expression:

$$K(\vec{x},\vec{\xi},p) = \begin{cases} A_1 K_1(\vec{x},\vec{\xi}) \\ A_2 K_2(\vec{x},\vec{\xi}) \\ .... \\ A_P K_P(\vec{x},\vec{\xi}). \end{cases} \tag{2}$$

Here, A_p are normalized (in the case for different registration systems or different dimension) coefficients, which are selected so that the noise dispersion of every imaging system is identical, and $K_p(\vec{x},\vec{\xi})$ is the point spread function of the imaging system with the number p.

2.2. Stabilizing restoration algorithm

For processing data, which have been formed by combined systems, we use the stabilizing algorithm [8]. This algorithm of object restoration is optimal into the mean square metric for any data discretization and it has an estimation accuracy. The stabilizing algorithm can be used even if Eq. (1) has no an unambiguous solutions. This stabilizing algorithm is applicable for combined systems with any kind of kernels (2).
We can describe the object by a series of an arbitrary system of basic functions

$$z(\vec{\xi}) = \sum_{k=1}^{\infty} c_k \psi_k(\vec{\xi}), \qquad \left|\vec{\xi}\right| \le \vec{S}. \tag{3}$$

Then the approximate solution of Eq. (1) is expressed as

$$z(\vec{\xi},\vec{\beta}) = \sum_{k=1}^{\infty} c_k(\vec{\beta})\psi_k(\vec{\xi}) = \sum_{m=1}^{\infty} \frac{\sum_{l=1}^{m} d_{lm}(\vec{\beta})\psi_l(\vec{\xi})\sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k, F)}{\beta_m + \sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k,\varphi_m)}, \tag{4}$$

where $\varphi_k(\vec{x},p) = \int_{-S}^{S} \psi_k(\vec{\xi})K(\vec{x},\vec{\xi},p)d\vec{\xi}$ are images of basic functions and $(\varphi_k,\varphi_l) = \sum_{x,p} \varphi_k(\vec{x},p)\varphi_l(\vec{x},p)$ are their scalar products. The summation is performed over all parameters of data of the image $F(\vec{x},p)$ discreditized for a input in the computer. The approximate coefficients of the expansion (3) are calculated as:

$$c_l(\vec{\beta}) = \sum_{m=l}^{\infty} d_{lm}(\vec{\beta}) \frac{\sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k, F)}{\beta_m + \sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k,\varphi_m)}. \tag{5}$$

Coefficients $d_{ln}(\vec{\beta})$ we can calculate by the recurrence relation

$$d_{ln}(\vec{\beta}) = -\sum_{m=l}^{n-1} d_{km}(\vec{\beta}) \frac{\sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k,\varphi_n)}{\beta_m + \sum_{k=1}^{m} d_{km}(\vec{\beta})(\varphi_k,\varphi_m)}, \tag{6}$$

where $l = 1,2,...,n-1$; $n = 1,2,...$. Here $d_{kk}(\vec{\beta}) = 1$, $k = 1,2,...$, and if $l > n$ then $d_{ln}(\vec{\beta}) = 0$.
The solution (4), (5) depends on the stabilizing vector parameter $\vec{\beta} = (\beta_1, \beta_2,...)$. Its optimal value is calculated from the minimum condition of the mean square error

$$\rho^2 = \left\langle \int_{-\vec{S}}^{\vec{S}} \left[z(\vec{\xi},\vec{\beta}) - z(\vec{\xi})\right]^2 d\vec{\xi} \right\rangle, \tag{7}$$

under statistical conditions: (a) an uncorrelated object $\langle c_k c_i \rangle = \langle c_k^2 \rangle \delta_{ik}$, (b) an uncorrelated object and noise $\langle c_k \gamma_i \rangle = 0$, (c) an uncorrelated noise $\langle \gamma_i \gamma_k \rangle = \gamma_*^2 \delta_{ik}$. Here γ_*^2 is the error dispersion in the case of the mean value of the data registration and the digitizing with zero mean value in determining $f(\vec{x},\vec{\xi},p)$. The brackets $\langle\ \rangle$ are the set realization averaging. The optimal values of the stabilizing parameter are labeled by *. For the determinated stabilizing parameters $\beta_m^* = \gamma_*^2 / \langle c_m^2 \rangle$ the minimum mean square errors can be written in the form:

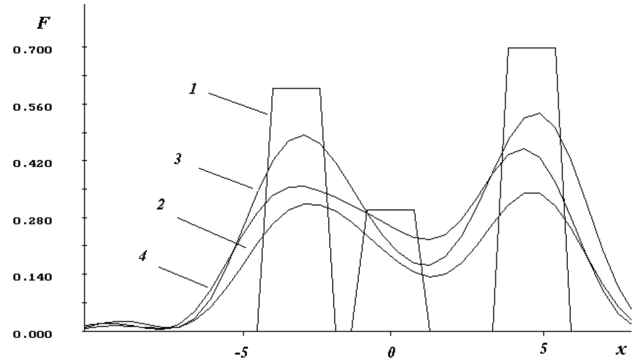

Figure 1: *Functions of the object and its images (1 – initial object $z(x)$, 2 - Image of the object when uniform illumination - $F_1(x)$, 3 - Image of the object with modulated illumination - $F_2(x)$, 4 - Image with modulated illumination - $F_3(x)$).*

$$\left\langle \Delta c_l^2(\vec{\beta}^*,\gamma_*) \right\rangle = \left\langle \left[c_l(\vec{\beta}^*,\gamma_*) - c_l\right]^2 \right\rangle = \sum_{m=1}^{\infty} \frac{d_{lm}^2(\vec{\beta}^*)\langle c_m^2 \rangle \gamma_*^2}{\gamma_*^2 + \langle c_m^2 \rangle \sum_{k=1}^{m} d_{km}(\vec{\beta}^*)(\varphi_k,\varphi_m)}, \tag{8}$$

$$\rho^2(\vec{\beta}^*,\gamma_*^2) = \sum_{m=1}^{\infty} \frac{\langle c_m^2 \rangle \gamma_*^2 \sum_{l=1}^{m} d_{lm}^2(\vec{\beta}^*)}{\gamma_*^2 + \langle c_m^2 \rangle \sum_{k=1}^{m} d_{km}(\vec{\beta}^*)(\varphi_k,\varphi_m)}. \tag{9}$$

Equations (8), (9) are evaluated an increase of the precision of the object determination of in the case of using the combined system in comparison with the case of one system. Also we can use the system information characteristic I_l, which is named as the information quality (IQ) in [9] about coefficient c_l as

$$I_l = -\ln \frac{\left\langle \Delta c_l^2(\vec{\beta}^*,\gamma_*^2) \right\rangle}{\langle c_l^2 \rangle} \tag{10}$$

The value $I = -\ln \rho^2(\vec{\beta}^*,\gamma_*^2) \big/ \left\langle \|z\|^2 \right\rangle$ is named [2] as the average information about the object.

3. NUMERIC SIMULATIONS

As usual, the object under research is the spatial distribution of the object reflection factor, the surface brightness distribution, or the absorption factor distribution into a volume. We consider numeric example to show possibilities of the image restoration algorithm.

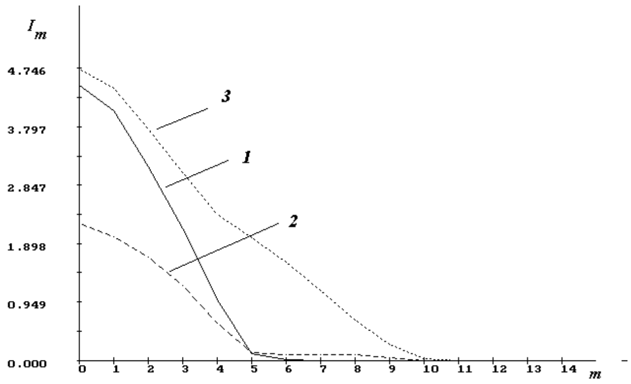

Figure 2: *IQ for cases with different illumination (1 - IQ with uniform object illumination, 2 - IQ with cosine illumination, 3 - IQ for the combined system).*

3.1. Diffraction limited system

The research object (Fig. 1) is the one-dimensional distribution of the object reflection factor, which we can describe by following equation

$$z(\xi) = 0{,}6\,\mathrm{Re}\,ct\frac{\xi+3}{2} + 0{,}3\,\mathrm{Re}\,ct\frac{\xi}{2} + 0{,}7\,\mathrm{Re}\,ct\frac{\xi-4{,}5}{3}, \qquad (11)$$

were

$$\mathrm{Re}\,ct\frac{x}{a} = \begin{cases} 1, & |x| \le a \\ 0, & \text{otherwithe } x \end{cases}$$

The diffraction limited imaging system with incoherent illumination can be described by PSF,

$$K(x,\xi) = \frac{\sin^2[a(x-\xi)]}{\pi a(x-\xi)^2}. \qquad (12)$$

where a is 0,8. Then the first image can be represented as

$$\int_{-S}^{S} z(\xi)\frac{\sin^2[a(x-\xi)]}{\pi a(x-\xi)^2}d\xi = f_1(x) + \gamma_1(x) = F_1(x), \quad |x| \le R. \qquad (13)$$

The second image is generated by the illumination (by the law) $E(\xi) = [1+\cos 2a\xi]$ of the spatial modulated illumination and it is written as

$$\int_{-S}^{S} z(\xi)[1+\cos 2a\xi]\frac{\sin^2[a(x-\xi)]}{\pi a(x-\xi)^2}d\xi = f_2(x) + \gamma_2(x) = F_2(x),$$

$$|x| \le R. \qquad (14)$$

The third image can be generate similarly

$$\int_{-S}^{S} z(\xi)[1+\sin 2a\xi]\frac{\sin^2[a(x-\xi)]}{\pi a(x-\xi)^2}d\xi = f_3(x) + \gamma_3(x) = F_3(x),$$

$$|x| \le R. \qquad (15)$$

According to Eq. (13) – (15), spatial Fourier spectra are zero outside of the cyclic frequency interval $|\omega| \le 2a$. Therefore, values of object spectrum are difficult for restoration just outside of this interval

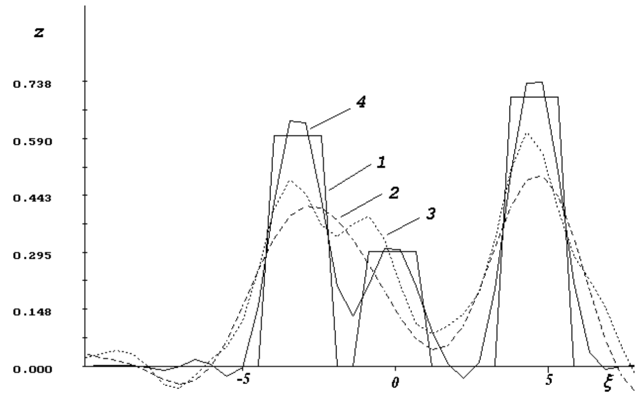

Figure 3: *Reconstructed object (1 – the initial object $z(x)$, 2 – reconstructed object with uniform illumination, 3 - reconstructed object with the modulated illumination, 4 - reconstructed object for combined system)*

restored. Let us consider the case of measuring the image in 40 equidistant points of reading x_i with the error dispersion $\gamma_*^2 = 10^{-3}$. The value of IQ about coefficients $\bar{c}_l$ of the complex Fourier series of the object can be calculated by the following equation [8]:

$$I_l = -\ln\frac{\left\langle\left|\Delta c_l^{(s)}\right|^2\right\rangle + \left\langle\left|\Delta c_l^{(c)}\right|^2\right\rangle}{\left\langle\left|c_l^{(s)}\right|^2\right\rangle + \left\langle\left|c_l^{(c)}\right|^2\right\rangle}. \qquad (10)$$

The data of the information analysis according to [7] are shown in Fig. 2. Integer values of $m(l)$ on the X-axis are proportional to the spatial frequency $\omega = m(l)\pi/S$, and along Y-axis there are values of calculated IQ. The object spectrum for frequencies $|\omega| \ge 2a$ can not be reconstructed because IQ about c_l with $l = 12$ and 13 ($m(l) = 6$) is zero. For modulated illumination case IQ increases within the interval $2a \le |\omega| \le 4\omega$, and substantially decreases for $|\omega| \le 2a$. The information curves (Fig. 2, curve 2) for both cases of the modulated illumination resemble. But, for the combined system, which consists of three image formation schemes under consideration, IQ increases substantially (Fig. 2 curve 3) in the frequency ranges considered and, therefore, as it is shown in Fig. 3 curve 4, the restoration results should be specified appreciably.

4. EXPERIMENTAL RESULTS

For the experimental confirmation of the numeric simulation results we take the one-dimensional object (Fig. 4c), which can be described by the equation to Eq. 3. The polymer worth equable scattering properties, which obey the Gauss law have been choose as the blurred media.

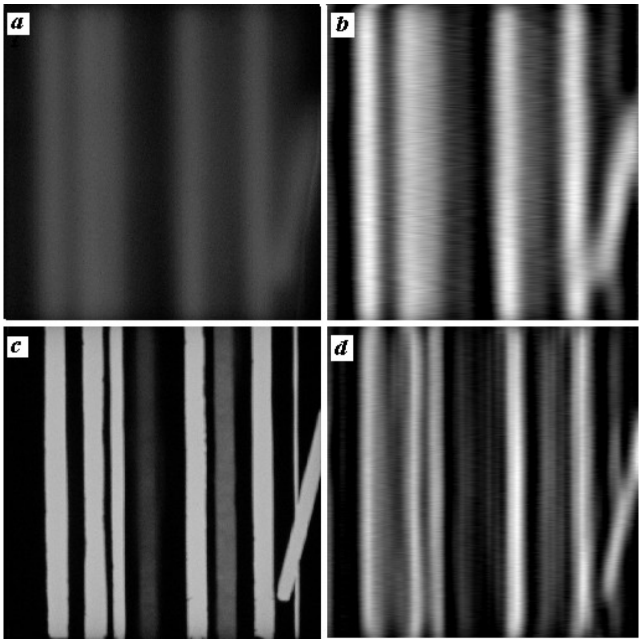

Figure 4: *Experimental results (reconstructed object by one image (a), blurred object (b), initial object (c), object reconstructed by the combined system (d)).*

In the image, blurred by such a way object, strips are not distinguishable (Fig. 4a). After restoration the object via one image (Fig. 4b) we can trace only some of the strips. In Fig. 4d the result of the new method application is presented. The restored image has a high contrast. The restoration results of the tilted strip (on the right part of images) show possibilities of the algorithm based on the forming several images outlines.

5. CONCLUSIONS

The optimized method of registration image processing has been developed. The numerical and physical experiments were performed for transferring the images through the scattering medias and diffraction limited imaging systems. It has been shown that the use of the several images of one object, which are different only by a type of its illumination, makes it possible to transfer its high spatial frequencies beyond the diffraction limit practically without degradation.

The method can be used for design of new devices for remote sensing, image communication and researches of physical objects.

6. REFERENCES

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