INFORMATIONAL ASSESSMENT OF SYSTEMS FOR SUPER-RESOLUTION RESTORATION FROM SEVERAL IMAGES

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ABSTRACT

The method of an assessment of quality of super-resolution restoration process from several images on the basis of application of an information theory is described in the paper. The significant advantage of the informational - theoretical approach is the opportunity of determination of total amount of information at once for the several imaging systems which register the same object, or for multiple registration of object in different conditions of illumination or with sub-pixel shift. Numerical calculations and physical experiments show that the using of the given approach makes it possible to predict theoretically optimal requirements of realization of transmission of images through scattering media. In the paper the results of the application of method quality assessment to two problems are given. In first the results of restoration from shifted images sequence depending on point spread function and number of shifts are brought. In the second example the quality assessment of real-time restoration of image from several noised and smoothed by motion of object during an exposure is made.

1. INTRODUCTION

Quality of isoplanatic (space invariant) imaging system can be evaluated by means of relation of Shannon information theory with minimum squared error (MSE) Wiener method of restoration [1]. In this work the informational estimation of isoplanatic optical systems, termed by amount of information in the image which can be used in view of possibilities of digital enhancement of registered images has been offered. Such approach makes it possible to optimize the design system for registration the image with the best result of restoration of images for visual perception [2].

In the nowadays imaging systems the focal plane arrays devices (CCD, CMOS, etc.) are used, thus the requirement of isoplanatism is broken (point spread function (PSF) and modulation transfer function (MTF) depends on shift of coordinates) [3]. Therefore we cannot use correctly the theory of isoplanatic systems for the comparative analysis of quality of the modern systems. Thus the theory for an evaluation of quality of the modern optical-electronic systems is not enough. In this paper the performance for evaluation of common class of linear system is offered. For convenience of practical using of the given evaluation the relation to isoplanatic analog – the amount of information in the image (Linfoot and Fellget) is considered. This approach has not received a wide practical spread. This fact is stipulated in our opinion by the following reasons: than more MTF on the given spatial frequency for the given signal to noise ratio (SNR) on the same frequency the channel capacity of system is more. Therefore, the comparative analysis of isoplanatic systems in view of possibilities of digital image processing for the given SNR in practice usually is easier to carry out only with using MTF.

For informational assessment [1] for optical-electronic system on the basis FPA we should take into account the essential dependence of restoration result of Wiener method on Nyquist limit in spatial frequencies [2].

Drawback of calculation MSE of restoration can be overcome by using the linear method of restoration [4], [5]. Given method can be used for non-isoplanatic (space-variant) imaging systems, and even in cases when for restoration irregularly sampled points are used [6]. Significant feature of application of such method is also the opportunity of restoration from several images simultaneously. And, as show experiments [7], the object can be restored from images which differ by physical conditions of formation. Formation conditions can have, for example, varying requirements of illumination, or to use multispectral radiation, smooth, caused by nonuniform curvilinear shift during an exposure of object, etc.

The considerable advantage of the informational-theoretical approach is the opportunity of determination simultaneously total amount of information for the several imaging systems registering the same object, or for multiple recording of object under various requirements of its illumination [8]. The total information, which calculates for process of registration of several images, or several systems of observation makes it possible to assess the dependence of restoration quality from parameters of system, PSF, illumination intensities. Also, the usage of the information performance helps to evaluate theoretically result of application of known methods [9], [10] of super-resolution restoration from sequence noised and blurred images and influence different parameters on results. Thus the amount of information for certain imaging system can be translated into the popular performance of optical system termed MTF. For non-isoplanatic systems, application MTF theory is not correct, therefore in this paper the definition of the equivalent MTF (EMTF) of non-isoplanatic optical-electronic system with the same amount of the information as isoplanatic systems is offered.

This paper is organized as follows. In Section 2 the mathematical model of process of image formation is given for the case when image captured by system with FPA.

In Section 3 the mathematical description of the theory for determination of amount of information and EMTF for such system is given.

Further (Section 4) results of numerical experiments showed opportunity of the using of information performance for two problems of restoration are given. The first ones illustrates efficiency of the approach for restoration from images sequence with a low-resolution, distinguished by sub-pixel shift. The second problem shows a possibility of increasing the quality of restoration of images smoothed by shift during an exposure time for simultaneously usage of several images smoothed in different directions.

In conclusion problems, which necessary to solve in the future are outlined.

2. OPTICAL-ELECTRONIC SYSTEMS

Let's consider the process of image gathering process (Fig. 1). The information about optical properties $Z(\vec{\xi},\nu)$ of object 1 is transferred by radiation through blurred media 2 with PSF into an optical system 3. The optical system 3 generates a monochromatic image $f(\vec{x},\nu)$ on a photosensitive array (e.g. CCD) 4 with the linear law

$$\int_{-\vec{S}}^{\vec{S}} Z(\vec{\xi},\nu) K(\vec{x},\vec{\xi},\nu) d\vec{\xi} = f(\vec{x},\nu), \qquad (1)$$

where $K(\vec{x},\vec{\xi},\nu)$ the kernel of the given integrating transformation, which in optics of imaging systems is named as PSF of system, parameter ν is temporal frequency of radiation illuminating array 4. The array 4 converts the image $f(\vec{x},\nu)$ into electrical signals $F_{i,j}$ by law (in the case of non coherence)

$$F_{i,j} = \iint_{A_{i,j}} Sen(\vec{x},\nu) f(\vec{x},\nu) d\vec{x} + \gamma_{i,j}, \qquad (2)$$

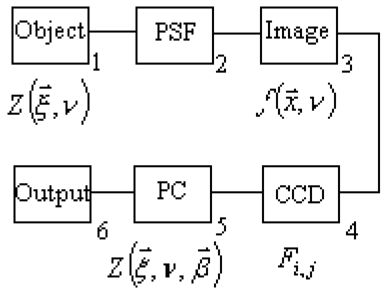


Figure 1. *The model of system for image registration and processing*

where integral takes area $A_{i,j}$ on photosensitive pixel with number (i,j), $Sen(\vec{x},\nu)$ is spectral function of sensitivity, $\gamma_{i,j}$ is error of an electrical signal $F_{i,j}$.

Combining expressions (1) and (2) we shall receive

$$\int_{-\vec{S}}^{\vec{S}} Z(\vec{\xi},\nu) K_1(i,j;\vec{\xi},\nu) d\vec{\xi} = f_{i,j} + \gamma_{i,j} = F_{i,j}, \qquad (3)$$

where $f_{i,j}$ is exact value of an electrical signal from an array device of the number (i,j), and

$$K_1(i,j;\vec{\xi},\nu) = \iint_{A_{i,j}} Sen(\vec{x},\nu) K(\vec{x},\nu) d\vec{x}.$$

Electrical signal $F_{i,j}$ enters the computer (the block of signals processing) 5 which uses their values for building the continuous variant $Z(\vec{\xi},\nu,\vec{\beta})$ required performance of $Z(\vec{\xi},\nu)$. We apply for realization this procedure the algorithm, which takes into account expressions (1), (2) or (3), and uses stabilizing parameter designated here by $\vec{\beta}$. Calculated by computer $Z(\vec{\xi},\nu,\vec{\beta})$ are represented by the device 6 in a user-friendly view. Quality of representation by the device 6 of calculated values $Z(\vec{\xi},\nu,\vec{\beta})$ depends on many objective and subjective parameters and it is not an aim of research in the given work. In this paper as well as in work [1] quality the system 1-5 is evaluated from positions of information theory but with usage of decomposition Kharunen-Loeva instead of decomposition of the required solution in system of eigenfunctions of kernel of the equation (1). Such approach takes into account non-isoplanatic devices of optical-electronic systems.

3. THEORY OF INFORMATIONAL ASSESSMENT

Quality of optical-electronic systems is defined in this paper by the maximum possible precision of restoration of object from electrical signals $F_{i,j}$. It is necessary to note, that a problem of definition of the continuous object from a finite number inaccurately known discrete values of a right part of the equation (3) is ill-posed problem [4], and can have a set of as much as different solutions. Therefore the additional information is necessary to check precision of its solutions by any method. The probability theory created for solutions of undetermined problems, makes it possible to calculate their probability characteristic. For its application object $z(\xi,\nu)$ and error $\gamma_{i,j}$ consider as random with *a priori* given statistical characteristic values. If random object and errors are characterized by density distribution of probability, than methods of probability theory makes it possible to calculate density distribution of probability of object restoration error of by any regularizing [4], [5] including a nonlinear method [11]. In this case it is possible to set the task: from a set of regularized methods to find such for which the probability of random object error to exceed the predefined value is minimal. Let for every spatial frequency of Fourier spectrum of object an user presets the minimal precision $a(\omega)$. Than we can to define the event

$$A(\omega,\vec{\beta}) \equiv : \{ \ \left|Z(\omega,\vec{\beta}) - Z(\omega)\right| < a(\omega) \ \},$$

which means, that precision of restoration of realization of object by certain of some regularized method $\vec{\beta}$ for given spatial frequency ω is satisfactory for user. Eigenvalue of amount of information in additional event

$$\overline{A}(\omega,\vec{\beta}) \equiv : \{ \ \left|Z(\omega,\vec{\beta}) - Z(\omega)\right| \geq a(\omega) \ \}$$

depends on restoration method, marked here by $\vec{\beta}$, statistical *a priori* information about random noise γ_{ij} and object $Z(\vec{\xi},\nu)$, from required by user precision $a(\omega)$, and from function K_1. Amount of information is found as

$$I(\omega,\vec{\beta}) = -\ln P\left[\overline{A}(\omega,\vec{\beta})\right], \quad (4)$$

where $P\left[\overline{A}\right]$ is probability of event $\overline{A}$. Obviously, that expression (4) is the natural performance of quality of system 1 - 5 (Fig.1) for application of the arbitrary restoration method $\vec{\beta}$. However another performance is more interesting for optimization of process of designing of optical-electronic systems

$$I(\omega,\vec{\beta}_o) = \max_{\vec{\beta}} I(\omega,\vec{\beta}) = -\ln P\left[\overline{A}(\omega,\vec{\beta}_o)\right]. \quad (5)$$

Here the maximum takes from the given set of all regularized restoration methods $\vec{\beta}$. Usually, for problem solution in real time, the set of restoration methods is specified in parametric view. The approximated solution of the equation (1) or (3) is of the equation (1) or (3) is represented as equation, depending on one or several parameters $\vec{\beta}$ which values are chosen with using of the additional statistical information about random object and noise so that whenever possible to minimize value $P\left[\overline{A}(\omega,\vec{\beta})\right]$. Thus, values of parameters are chosen so to satisfactory results for the user as often as it is possible. More simply values $P\left[\overline{A}(\omega,\vec{\beta}_o)\right]$ and optimal operator $\vec{\beta}_o$ are determined, if to assume, that coefficients of decomposition of random object on system of eigenfunctions of the equation (1) are statistically independent. Then the functional $\vec{\beta}$ is represented by set of monotonic functions. For this case in work [5] the expression for a probability density function of restoration error on the given spatial frequency is represented depending on an unknown monotonic function [5]. Search of this function was carried out numerically from a requirement $\min P\left[\overline{A}(\omega,\vec{\beta})\right] = P\left[\overline{A}(\omega,\vec{\beta}_o)\right]$ with using of obvious relation from [5]. Two sets of functions were explored a set of symmetric functions $\vec{\beta}_{lin}$ and set of monotone functions $\vec{\beta}$. It is shown, that under a natural requirement of major uncertainty in coefficients of object decomposition, when their function of distribution of density of probabilities considerably differs from pulsing (delta functions) the minimum from a set of linear functions exceeds minimum from all monotonic functions only a little, namely

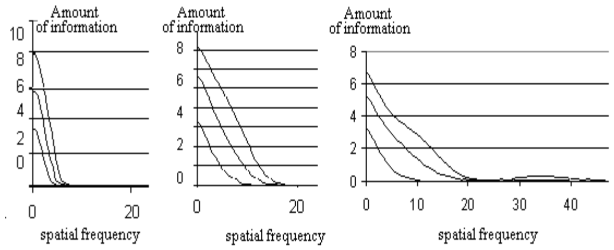
$$\frac{P\left[\overline{A}(\omega,\beta_{0lin})\right] - P\left[\overline{A}(\omega,\beta_0)\right]}{P\left[\overline{A}(\omega,\beta_{0lin})\right]} \leq 0.15 .$$

Thus, separated computing results, and mainly used in practice the incomplete statistical information about object and noise as their correlation functions of the second order determine actuality of information assessment of optical-electronic systems with using of set of the linear restoration methods.

4. EXAMPLES OF USING OF INFORMATIONAL CHARACTERISTIC

4.1 Restoration from images sequence

The problem of restoration of object from its several blurred images was numerically simulated. The analysis of behavior of the information in dependence on



(a) (b) (c)

Figure 2. *The dependence of amount of information on blurs. Upper curves correspond to 100 images, middle to 10 and bottom to one.*

number of images and PSF is carried out. Sensitivity of pixel of focal-plane array was considered as the uniform with array fill factor 50%. The period of array is dimensionless and has taken as 0,08. Spatial frequency is also dimensionless.

In Figure 2 the dependence of the amount of information on spatial frequency for cases of the heavy (a), mean (b) и small (c) blurs (half widths of PSF are about 10 pixels, 1 pixel and 0,23 of size of pixel respectively) of the initial one-dimensional image is shown.

When blur is heavy the increasing of number of sampled points in the image gives enhancement of results and SNR only on low spatial frequencies [12].

Results of two-dimensional images processing are shown in Figure 3. The considerable improving of visual quality when we use a great number of images. In this case the object is the picture 256x256 pixels, blurred images were received as images 12x12 pixels.

4.2 Restoration of smoothed images

4.2.1 Theory of restoration from several smoothed images

Let's consider the problem of restoration of object from its several images smoothed in different directions, in particular, from two images, smoothed in x and y axes. Initial object we can represent as:

$$Z(\xi,\eta)=\sum_{p}\int_{-R}^{R}\int_{-R}^{R}F_p(x,y)Q(x-\xi,y-\eta)dxdy \quad , p=1.2$$

For many cases of processing smoothed images it is necessary to have the algorithm, which works with high speed to process images in real-time mode. Therefore the "sliding window" algorithm has been used. Let the width of smooth is equal $2N+1$ points. Then restoration

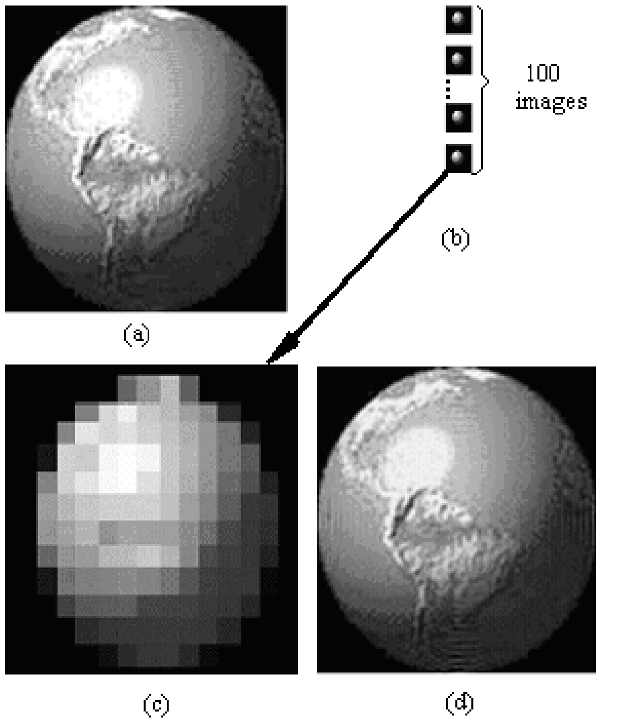


Figure 3. *Result of restoration. (a) - initial image; (b)-images received from (a) by shift, blur and sampling;(c)- enlarged image (b); (d) - restored from 100 images.*

of object point $Z(\xi_i,\eta_j)$ comes to summation with filter:

$$Z(\xi_i,\eta_j)=\sum_{p,l,m}F_p(x_{i+l},y_{j+m})Q(x_{l+N},y_{m+N}) \quad p=1,2;$$

$$l=-N,...N;\; m=-N,...N.$$

The extreme points of object which are placed apart from boundaries in a distance smaller than smooth are not processed. The approach is easily extended for the case of various size of smooth in the registered images.

In general case algorithm demands as a basis set of functions a complete set of orthonormalized functions. As a rule, it is trigonometric series. However for usage of the given approximate "sliding window" algorithm the contribution odd component is equal to zero, and as certain realization the system has been selected

$$\psi_1(\xi)=1/\sqrt{2S},\;\; \psi_l(\xi)=\cos(\pi l\xi)/\sqrt{S},\;\; l=2,.... \quad (6)$$

Two-dimensional basis functions as multiplication of one-dimensional functions (6):

$$\psi_{lm}(\xi,\eta)=\psi_l(\xi)\psi_m(\eta) \quad (7)$$

4.2.1 Numeric simulation

For an evaluation of possibilities of restoration for the given filter and determination of required filter sizes in

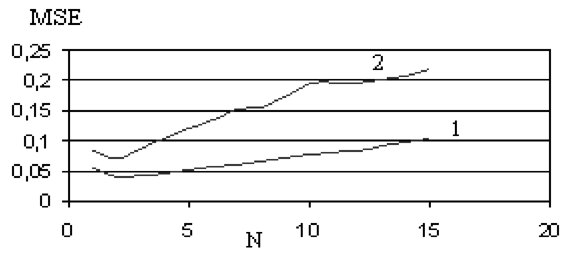


Figure 4. *The dependence MSE of restoration by the filter 40x40 points from a half-size of smooth N. A curve 1 - restoration from one image, 2-from two.*

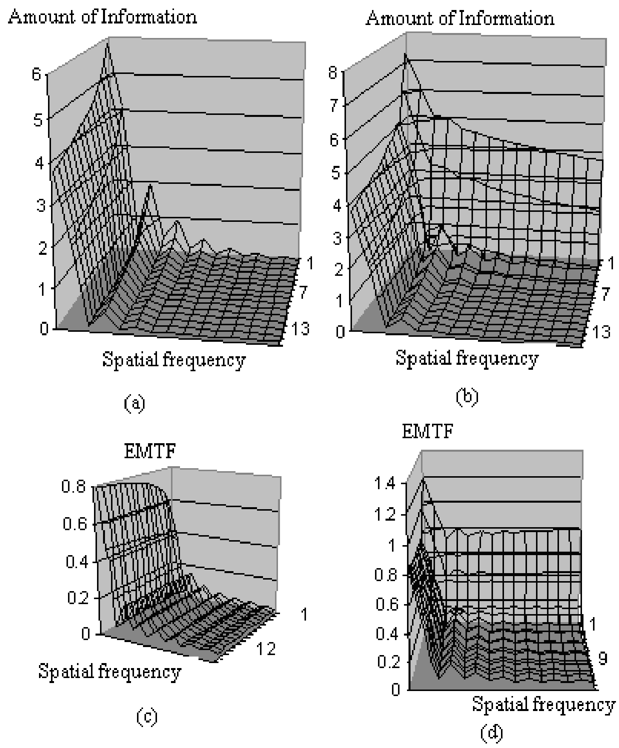


Figure 5. *Amount of information and EMTF as functions from dimensionless spatial frequencies* (l,m). $N=10$. *(a, c)-amount of information and EMTF for restoration from one image; (b, d) – for two.*

comparison with width of smooth the calculations MSE (Fig. 4) are performed.

As shown in this example, the first curve has a flat site in field when the size of the filter approximately is two-fold more than smooth size. This relation frequently used in practice. The quality of restoration from two images is considerably better, than from one.

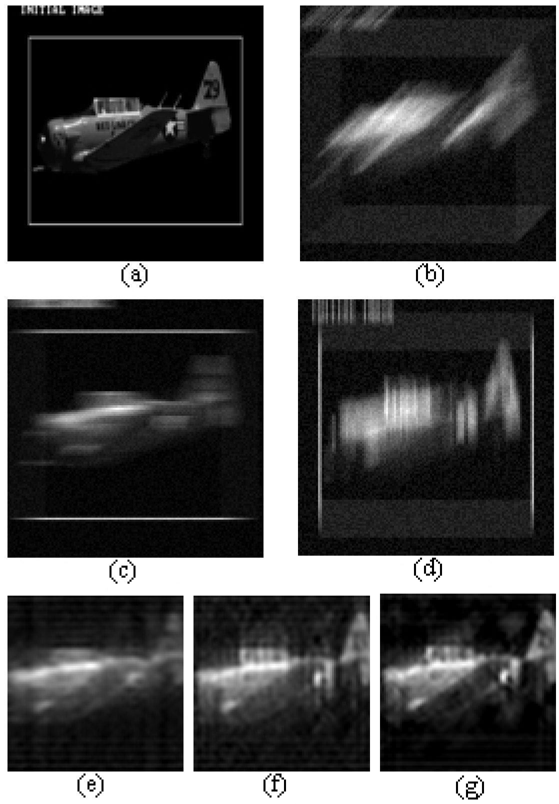


Figure 6. *(a) – initial object,(b) – smoothed in diagonal direction,(c, d) – smoothed horizontally and vertically images, (e), (f) and (g) -– restored from one, two and four smoothed images by real time "sliding window" algorithm.*

For assessment of quality of restoration for such system of observation in the frequency field the calculations of amount of information, statistically defining quality of processing were used. The given value shows the quality of restoration for the spatial frequencies correspondent to basis functions with a coefficient (l,m) (Fig. 5).

In figure 5 is shown, that the amount of information strongly depends on coordinate of shift, thus oscillating I deals with EMTF on k frequency by relation:

$$T_k = \sqrt{\gamma^2 (\exp I_k - 1) / \left\langle |c_k|^2 \right\rangle},$$

γ^2 is SNR, $\left\langle |c_k|^2 \right\rangle$ - *a priori* specified values described kind of restored objects.

Behavior of EMTF (Fig. 5 (c, d)) is similar to behavior of the amount of information, but it can differ by presence of large oscillations. Blurred images processing shows actual improving of visual quality for usage of two and four smoothed (with 10% of noise) images (Fig. 6).

5. CONCLUSIONS

In the paper the method of assessment of quality of systems for super-resolution restoration from several images on the basis of application of information theory is described. The possibility of determination of amount of information for multiply object registration is shown under its requirements smooth and sub-pixel shift. Given information characteristic was used before [7], [8] to transfer high spatial frequency of images on low frequencies. At present the solution of problems of precise registration [13] and exact determination of sub-pixel shift in conditions of blur and noise in images are important.

6. REFERENCES

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